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Surface Mass-Transfer Correlations

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Nomenclature

C_p = specific heat at constant pressure
F = mass-injection ratio, $(\rho v)_c/(\rho u)_1$
G = nondimensional injection variable, $C_{p_c}F/C_{p_1}St_0$
m = molecular weight
n = number of molecules per unit volume
q = surface heat flux
St = Stanton number
T = temperature
u = velocity along the surface
v = velocity normal to the surface
Z = corrected molar injection rate, $0.6\ m_1C_{p_1}G/m_cC_{p_c}$ or $0.6\ n_cv/n_1u_1St_0$
ρ = density

Subscripts

a = adiabatic
c = coolant
0 = zero injection condition
w = surface value
1 = freestream value

Bartle and Leadon[1] have reported experimental turbulent mass-transfer cooling data obtained at a Mach number of 3.2 using nine different coolants. Having found that no extant turbulent boundary layer analysis adequately predicts the heat transfer reductions measured and that the use of the cooling effectiveness instead of the Stanton number reduction, St/St_0, circumvents the "cranky nature" of the latter and "minimizes the effect of Reynolds number," they recommend that the engineer make direct use of their empirical effectiveness formula. The local heat-flow reduction, q/q_0, then is to be obtained readily from Eq. (1):

$$\text{effectiveness} = \frac{T_w - T_c}{T_{aw0} - T_c} = \left(1 + \frac{C_{p_c}F}{C_{p_1}St_0}\,\frac{q_0}{q}\right)^{-1} \quad (1)$$

Tewfik[2] criticizes the conclusions of Ref. 1, pointing out that the effectiveness depends strongly on the wall-temperature level near adiabatic conditions. Although Tewfik's comments are not incorrect, they miss the mark. The engineer, to whom the recommendations of Bartle and Leadon are addressed, is interested in injection cooling for the very purpose of making the wall temperature much different from the adiabatic wall temperature. Effectively, Tewfik finds no serious fault with the recommendations in this regime.

It should be pointed out that there is, in fact, a basic objection to some of the conclusions of Ref. 1. It is suggested therein that the experimental data correlate much better when presented as effectiveness rather than as heat-flow reduction. It is clear from Eq. (1), however, that when the effectiveness is small as compared with unity (as it is for much of the experimental measurements) the percentage scatter in the two quantities is inherently almost the same.

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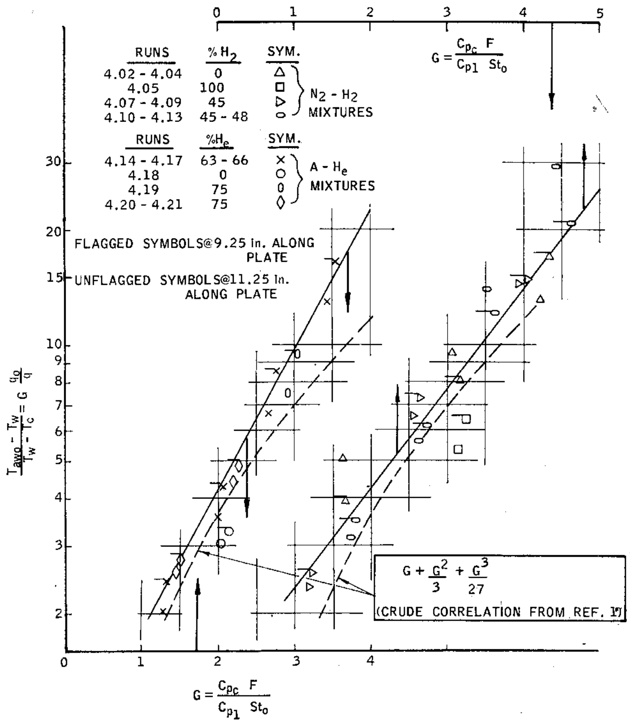

Fig. 1 Corrected heat-flow reduction for monatomic and diatomic gases

What may have been in mind in presenting a single correlating curve for the effectiveness of all nine coolants is that it need not be known too precisely for some engineering purposes when it is small. Be that as it may, it is quite misleading to suggest that the heat-flux reduction itself is evaluated properly by using such a crude correlation.

Take a closer look at the portion of the heat-flow reduction data covering monatomic and diatomic gas injection (66% of the total). The monatomic gas injection data are plotted at the left in Fig. 1 and the diatomic, at the right. The latter has two salient features: 1) the data scatter greater at 11.25 in. along the plate than at 9.25 in., and 2) the lack of a significant trend as the composition of the injected gas changes from pure nitrogen to pure hydrogen. As can be seen, giving special emphasis to the data at the earlier location allows a single straight correlating line to be drawn. The corresponding correlation for monatomic gas injection is shown at the left. By way of contrast, the dashed lines present the crude correlation of Bartle and Leadon.

In the context of the basic study of Ref. 3, the single correlation for all the diatomic gas injection data indicates that, in this operating regime, i.e., high molar injection rates into turbulent flow, injection fluids achieve the same corrected heat-flow reduction q/Gq_0 when their molar injection rates are the same even though the fluids themselves are not similar thermophysically. Consistent with this finding, the slope of the correlating line for monatomic gas injection is exactly 1.4 times that for diatomic gas injection. These results suggest that the volumetric displacement mechanism of mass transfer for achieving surface effects equivalent to the slip and temperature-jump characteristics of low-density fluid mechanics is more effective in turbulent flow than in laminar flow.

The data correlations for monatomic and diatomic gas injection may be unified into a single correlation of the heat-flow reduction as a function of the corrected molar injection rate Z as Eq. (2):

$$q/q_0 = 1.3(m_1\,C_{p_1}/m_c\,C_{p_c})^{1/2}Ze^{-Z} \qquad (Z > 1) \quad (2)$$

In view of the scatter of the data, however, there is some question as to whether one is justified in using a more detailed correlation than Eq. (3):

$$q/q_0 = 1.4Ze^{-Z} \qquad (Z > 1) \quad (3)$$

In any case, the principal long-term value of these and related data may well be other than the establishment of mass-transfer cooling levels. In the long view, they may be even more valuable as a new type of observation of the attributes of turbulence near a surface.

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Method of Characteristics and Velocity of Sound for Reacting Gases

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ALTHOUGH there is an evident disagreement between the definition of the sound speed in a reacting gas given by Resler,[5–7] and that found by Kirkwood, Chu, and others, the reason for this discrepancy has not been made clear. In this note, the author will show how Resler's analysis may be revised to become consistent with the classical analysis of the method of characteristics. Accordingly Eqs. (19) and (20) of Resler's paper,[7] describing the two-dimensional steady flow of a gas, will be considered. These may be written

$$\theta_n + \rho_s/\rho + w_s/w = 0 \qquad (1)$$

$$\theta_s + p_n/\rho w^2 = 0 \qquad (2)$$

$$ww_s + p_s/\rho = 0 \qquad (3)$$

where θ is the flow angle, w the velocity, p the pressure, and the subscripts s and n denote differentiation with respect to the stream direction and its normal, respectively. Eliminating w_s between Eqs. (1) and (3) and introducing the directional derivative

$$\theta' = \theta_s + \theta_n/\beta \qquad (4)$$

where $\beta = (M^2 - 1)^{1/2}$, $M = w/a$, and a is to be determined, Eqs. (1–3) are combined to give

$$\theta' + \beta p'/\rho w^2 = 1/\beta\rho(p_s/a^2 - \rho_s) \qquad (5)$$

Since Eq. (5) must contain only primed derivatives when $dn/ds = 1/\beta$ is a characteristic direction, Resler set the right-hand side equal to zero in order to define the velocity of sound and obtained $a = (p_s/\rho_s)^{1/2}$.

Now consider the classical approach in which the characteristics are regarded as surfaces along which the derivatives are indeterminate. Assume further that the pressure is given by the simple functional relation

$$p = p(\rho, \eta) \qquad (6)$$

with η satisfying a reaction rate equation of the form

$$w\eta_s = \varphi \qquad (7)$$

where φ does not depend on derivatives of the flow quantities. After eliminating w_s from Eqs. (1) by Eq. (3) and using Eqs. (6) and (7), one obtains

$$\theta_n + \beta^2\sigma_s/M^2 = a_1\varphi/w \qquad (8)$$

$$\theta_s + \sigma_n/M^2 + a_1\eta_n = 0 \qquad (9)$$

where $M^2 = w^2/(\partial p/\partial\rho)$ and $a_1 = (\partial p/\partial\eta)/\rho w^2$ have been substituted, and $\sigma = \log\rho$.

Let the equation for the characteristic be given by $n = n(s)$ Then Eqs. (8) and (9), with the conditions

$$\sigma_s + \sigma_n n' = \sigma' \qquad (10)$$

$$\theta_s + \theta_n n' = \theta' \qquad (11)$$

$$\eta_n n' = \eta' - \varphi/w \qquad (12)$$

constitute a system of linear equations in the derivatives of σ,θ, and η. By using Kramer's rule and applying the condition of indeterminancy, the differential equations for the characteristics are obtained as

$$n' = 0 \qquad n' = \pm 1/\beta$$

along which are the compatibility conditions,

$$\eta' = \varphi/w \qquad (13)$$

and

$$\pm\theta' + \beta(a_1\eta' + \sigma'/M^2) = a_1M^2\varphi/\beta w \qquad (14)$$

respectively. Combination of the partial derivatives of p in Eq. (14) gives

$$\pm\theta' + \beta p'/\rho w^2 = (\varphi/\rho a^2\beta w)(\partial p/\partial\eta) \qquad (15)$$

The significant difference between Eq. (15) and Resler's corresponding result is the inhomogeneous term on the right-hand side. Now reconsider Eq. (5) and assume for p, the pressure relation of Eq. (6). After using Eq. (7) and simplifying, the right-hand side of Eq. (5) becomes

$$(1/\rho\beta)[(\partial p/\partial\rho)/a^2 - 1](\partial p/\partial s) + (\varphi/\rho a^2\beta w)(\partial p/\partial\eta)$$

In order for the prime differentiation to be in the characteristic direction, the coefficient of ρ_s must vanish. Thus is obtained

$$a^2 = (\partial p/\partial\rho)\eta$$

which is the "frozen" velocity of sound. In addition the inhomogeneous term in agreement with Eq. (15) is obtained. Thus it appears that setting the right-hand side of Eq. (5) equal to zero to define the velocity of sound is too severe a restriction.

From the preceding analysis the following conclusions can be drawn:

1) The introduction of finite reaction rate equations leads to the addition of inhomogeneous terms to the differential equations along the characteristic directions.

2) Contrary to Resler's assumption, it is not possible to define the velocity of sound using the equations of continuity and momentum alone. The reaction rate equations as well as the pressure relation must be used.

3) The velocity of sound for the method of characteristics is always that calculated for frozen flow conditions in agreement with Refs. 1–3 and 8–10.

One reason that the frozen speed of sound is not observed frequently is that the medium with finite reaction rates is dispersive and the bulk of the energy may not coincide with the wave front. The highest frequencies propagate with a velocity near the frozen speed of sound and the lowest frequencies with a velocity near the equilibrium speed of sound.

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